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An interpretable intuitionistic fuzzy inference model for stock prediction

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ABSTRACT

Stock price prediction and modeling demonstrate high economic value in the financial market. Due to the non-linearity and volatility of stock prices and the unique nature of financial transactions, it is essential for the prediction method to ensure high prediction performance and interpretability. However, existing methods fail to achieve both the two goals simultaneously. To fill this gap, this paper presents an interpretable intuitionistic fuzzy inference model, dubbed as IIFI. While retaining the prediction accuracy, the interpretable module in IIFI can automatically calculate the feature contribution based on the intuitionistic fuzzy set, which provides high interpretability of the model. Also, most of the existing training algorithms, such as LightGBM, XGBoost, DNN, Stacking, etc, can be embedded in the inference module of our proposed model and achieve better prediction results. The back-test experiment on China's A-share market shows that IIFI achieves superior performance — the stock profitability can be increased by more than 20% over the baseline methods. Meanwhile, interpretable results show that IIFI can effectively distinguish between important and redundant features via rating corresponding scores to each feature. As a byproduct of our interpretable methods, the scores over features can be used to further optimize the investment strategy.

1. Introduction

With the continuous development of the social economy, various investment financial instruments have emerged. Stock is one of the most active and widely used financial products. Appropriate stock investment can quickly obtain a large amount of income. Therefore, the trading and forecasting of the stocks have also become one of the topics of widespread concern.

Predicting the movement of stock prices is a complex issue. According to the inference of efficient market hypothesis (EMH) (Fama, 2021), the current market is still predictable. The stock price is generated through continuous matching transactions between the buyer and the seller. There are many potential predictable components in trading information (Cooper, 1999; Lehmann, 1990). Different researchers have proposed many different types of stock investment predicting methods.

Econometric model is a kind of methods to predict stock valuation. There are special prediction models for financial time series, such as Arima, ARCH, and GARCH (Gencay, 1996; Idrees, Alam, & Agarwal, 2019; Khashei & Bijari, 2010). This type of model predicts the volatility of stock prices and further predicts the prices and returns based on this. However, such models often have more pre assumptions and constraints, and cannot describe the short-term rapid fluctuations in the stock market.

Multiple factors need to be used to judge the stock and market situation. The information and data released by companies and governments is important information. Technical analysis means that technicians construct indicators by observing the trend of market transaction data and relying on the law of exploration. Common classical indicators include random index (KDJ), relative strength index (RSI), trend index (DMI), boll line (BOL), deviation rate (bias), smooth similarity, and difference average (MACD), etc. It has become a complex problem to judge the effectiveness of various indicators and eliminate the correlation among them. The timing of buying and selling operations need to be artificially judged. However, this method lacks effective test methods and appropriate generalization space. Moreover, it is susceptible to the rational or irrational subjective influence of traders.

In recent years, the prediction technology of artificial intelligence models has made a significant breakthrough. Compared with the traditional linear regression method, artificial intelligence algorithm has

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strong processing ability for high-dimensional, low signal-to-noise ratio, nonlinear and non-stationary data. It has good data adaptability. It has been widely used in the financial field. Neural network methods have a strong nonlinear fitting capability and can theoretically map any complex nonlinear relationship. It has strong robustness, memory ability, nonlinear mapping ability, and strong self-learning ability. Artificial intelligence models can adaptively perceive features and combine them, but its perception process is black-box behavior that does not intuitively show the correlation with prediction results. Although Artificial intelligence models can achieve more accurate prediction results, their inference process is not easily explained. Many scholars have made many explorations to allow humans to understand the learning mechanisms of Artificial intelligence models. A new research area called explainable AI Samek, Montavon, Vedaldi, Hansen, and Müller (2019) has emerged to address this problem. (Lipton, 2018) states that a clear picture of the complete computational process of the model is shown compared to similar linear functions. It is more important for the user to have trust with the model and the causality of the input data and the results. In the scenario of stock trading, where real financial transactions are involved, traders and modelers want to have sufficient knowledge of model interpretability.

The concept of fuzzy set (FS) was proposed by (Zadeh, 1965) in 1965, and then the concept of fuzzy inference system was gradually developed. Fuzzy inference system is a system in which input, output, and state variables are defined on fuzzy sets, which is a generalization of deterministic systems. In a deterministic system, once the state and input at a specific time are determined, the state and output of the next time are clearly and uniquely determined. The stock price, which will be affected by many factors, is not a deterministic system. It is an appropriate choice to use fuzzy sets for representation.

The combination of neural network and fuzzy inference system is called Neuro-Fuzzy System (NFS). This significantly improves the accuracy of the fuzzy inference system. Neuro-Fuzzy system can imitate human comprehensive inference to deal with fuzzy information processing problems that are difficult to solve by conventional mathematical methods. It can solve nonlinear problems well and has been widely used in automatic control, pattern recognition, decision analysis, time series signal processing, man–machine dialogue system, economic information system, medical diagnosis system, earthquake prediction system, weather prediction system, etc. (Vieira, Dias, & Mota, 2004)

Neuro-fuzzy system is composed of fuzzy rules based and fuzzy inference engine. There are mainly two kinds of common fuzzy inference systems (Vieira et al., 2004). Mamdani type pays attention to intuitive interpretability, while Takagi–Sugeno–Kang type has higher computationally efficiency. Neuro-fuzzy systems rely on fuzzy rules to build interpretability. Neuro-fuzzy system based on fuzzy rules has the problem of high time complexity, even when faced with a simple problem. As the number of features increases, the antecedent part of fuzzy rules becomes complex and loses interpretability. This makes the neuro-fuzzy system unable to receive a large number of features.

In order to enable investors to understand the learning mechanism, fuzzy systems are used (Cao, et al., 2020; Rajab & Sharma, 2019; Xie, Rajan, & Chai, 2021). The inference process of fuzzy systems is based on linguistic rules. It is more interpretable than artificial intelligence algorithms. Because of the dependence of neuro-fuzzy systems on fuzzy rules, as the number of features increases, the number of parameters to be learned will increase exponentially and the interpretability will be lost. A large number of input features cannot be accepted in neuro-fuzzy systems. Although the artificial intelligence model has high prediction accuracy, it lacks interpretability.

In stock forecasting, it is not necessary to have complete knowledge of the forecasting process as in the case of linear models. (Jacovi & Goldberg, 2020) points out that there are only two questions about interpretability that need to be addressed while ensuring high accuracy. Which features have a large impact on the prediction results. And, how to optimize the model through interpretation. For traders, knowing which features have a greater impact on the results can help determine market conditions and the effectiveness of strategies. Interpretable model results allow traders to make certain recommendations for strategy optimization. In addition, for modelers, the interpretable results allow for further optimization of the prediction model.

Motivated by the problems mentioned above, this paper proposes an interpretable intuitionistic fuzzy inference model (IIFI). IIFI aims to predict stock prices more accurately and construct interpretable evaluations of feature importance and the impact of features on the results. The main contributions of this study are as follows.

1. An indivisible six-layer neuro-fuzzy network model is proposed, which can be updated on-line. It solves the problem of the exponential growth of parameters that need to be learned. And this paper uses the data-based unsupervised fuzzy set construction method, which can apply the data set more effectively and avoid human influence.
2. Since the input and output of the prediction process take place in the fuzzy set space, we can evaluate the contribution of each feature to the output result and form a visual result by adjusting the intuitionistic fuzzy set information of the input. The interpretability module in the proposed model can quantify the contribution and negative contribution of each feature to the prediction result.
3. We conduct experiments on the Chinese A-share market data set. The proposed IIFI is superior to existing artificial intelligence algorithms and neuro-fuzzy systems. At the same time, the predictions of IIFI can be explained, which can intuitively express the importance of each feature and be used for feature selection and evaluation.

The rest of this paper is organized as follows. For completeness, Section 2 introduces the related work of stock prediction. Section 3 the interpretable intuitionistic fuzzy inference model is proposed and presented. Experiment results are analyzed and discussed in Section 4. Finally, Section 5 concludes and outlines future works.

2. Related work

2.1. Neuro fuzzy systems and its application in stock prediction

Neuro-fuzzy system (NFS) is generally based on the combination of neural networks and fuzzy systems. It combines the learning ability of artificial neural network and the semantic knowledge representation of fuzzy system. The core of fuzzy system is fuzzy set. The fuzzy sets theory was initiated by Zadeh (1965) in 1965. Fuzzy set is a linguistic variable with semantics.

Fuzzy systems use a series of fuzzy rules to express decisions. According to the difference of fuzzy rules, fuzzy inference system can be divided into two types: Mamdani type and Takagi–Sugeno–Kang(TSK) type (Shihabudheen & Pillai, 2018).

Mamdani Rule: $if\ x_1\ is\ A_1\ and\ x_2\ is\ B_1\ then\ y_m\ is\ Y_1$

TSK Rule: $if\ x_1\ is\ A_1\ and\ x_2\ is\ B_1\ then\ y_{TSK} = p_1x_1 + q_1x_2 + r_1$

In the above equation, x_1 and x_2 are the input linguistic variables. A_1 and B_1 are the fuzzy sets. y_m is the output linguistic variables and Y_1 is the fuzzy set. x_1, x_2 and y_m can also be expressed as membership values. y_{TSK} is a numerical value that is calculated from a linear function of input. p_1, q_1 and r_1 are the parameters of the linear function and they are the objects to be trained. The difference between Mamdani-type fuzzy inference rules and TSK-type fuzzy inference rules is the conclusion part of the rule. If a crisp result is desired, the Mamdani-type fuzzy rule also requires a defuzzification operation. The fuzzy rules of the former are shown in the form of fuzzy sets, while the fuzzy rules of the latter are represented by linear sub-models. The former pays attention to intuitive interpretability, while the latter has higher computationally efficiency.

The fuzzy inference mechanism consists of 3 stages. In the first step, the crisp input number will convert into fuzzy numbers by the membership function of fuzzy sets. The degree to which the clear input data belongs to the fuzzy set is determined by the membership function in a one-to-many manner. The second step will build fuzzy inference rules. The third stage is the inverse operation of the first stage, which converts the obtained fuzzy number into a crisp value, which is generally called defuzzification. Different researchers will use different combinations of methods to build NFS, but generally all have the above three steps.

Fuzzy inference systems propose a method based on mathematical calculus to transform human subjective knowledge of real processes. The addition of artificial intelligence algorithms makes the process more precise. Adaptive-network-based fuzzy inference system (ANFIS) is the most commonly used neuro-fuzzy system based on TSK inference system (Jang, 1993). ANFIS has five layers. ANFIS divides the parameters that need to be learned into two categories. ANFIS used two different learning methods to determine parameters. NFS was used in many fields as an effective tool and in many cases has advantages over classical technique (Vieira et al., 2004). Alizadeh, Rada, Jolai, and Fotoohi (2011) used the fundamental properties of indices in the stock market as input to construct portfolios through forecasts based on the ANFIS method. On the basis of using ANFIS for stock prediction, Esfahanipour and Aghamiri (2010) used the fuzzy C-Mean clustering implemented for identifying number of fuzzy rules, which improves the accuracy. Mohamed, Ahmed, Mehdi, and Hussain (2021) used the fuzzy rules obtained by ANFIS to establish the ranking of feature importance in the stock forecasting problem.

In order to improve the accuracy of the membership function, the input features were preprocessed by wavelet change (Artha, et al., 2018; Chandar, 2019a). Sharma, Hota, and Rababaah (2021) proposed a prediction model combining ANFIS and wavelet transforms that verified the influence of the number and type of membership functions in the prediction process.

In some fuzzy system models, the establishment of membership function relies too much on expert knowledge. The establishment of membership functions in an unsupervised manner is more suitable for most scenarios. Kumar, Jain, and Singh (2021) and Chandar (2019b) adopted the method of subtractive clustering to determine the membership function and cluster center. Stock price data is typical time series data. In order to take advantage of this more fully, Mahmud and Meesad (2016) and Xie et al. (2021) had proposed two new architectures for NFS models. They pass the time series into the model as information to get more accurate results. The main drawback of ANFIS is that it is computationally intensive and generates complex models for even relatively simple problems. At the same time, when the number of features increases, a large number of fuzzy rules will be generated, which reduces the interpretability of fuzzy rules. Nair, Minuvarthini, Sujithra, and Mohandas (2010) and Nair, Dharini, and Mohandas (2010) used decision trees for feature screening. The filtered features were used as the input of the NFS model to reduce the complexity of NFS. Kasabov and Song (2002) and Tung, Quek, and Guan (2011) classified the training data and train their respective NFS. This reduced the complexity of fuzzy rules for each individual classification.

However, these methods do not completely solve the disadvantage of neuro fuzzy system. As the number of features increases, computational efficiency and interpretability will drop. In this paper, we propose a model to construct fuzzy rules based on intuitionistic fuzzy sets and contribution degrees, and the model parameters grow linearly with the increase of features. Contributions and negative contributions of features will be calculated, which provides model interpretability in place of fuzzy rules.

2.2. Application of artificial intelligence algorithms in stock prediction

Stock price data is one of the most important data in stock forecasting. Stock price data is a typical financial time series data. The correlation and modeling of financial time series is a challenging problem. In the past decades, machine learning and deep learning methods have become very popular in financial time series modeling. They tend to have higher accuracy than traditional financial econometric model methods such as ARCH (Alam, Siddikee, & Masukujjaman, 2013) and GARCH (Dana, 2016). Traditional econometric methods are often based on linear models, feature mining relies on the experience of traders, and the features that can be considered are limited.

Li, Turkington, and Yazdani (2020) explored the influence of features on the labels of stock price prediction results in machine learning models and pointed out that there are at least three correlations between stock-related factor feature data and returns. Contributions from linear relationships, nonlinear relationships, and interactions between factors. Machine learning methods have good nonlinear mapping capabilities and the ability to handle a large number of features, such as random forests (Patel, Shah, Thakkar, & Kotecha, 2015), decision tree (Basak, Kar, Saha, Khaidem, & Dey, 2019; Sun, Liu, & Sima, 2020), artificial neural networks (Ballings, Van den Poel, Hespeels, & Gryp, 2015), support vector machines (Ballings et al., 2015), etc. Traditional support vector machines cannot accurately describe the aggregation phenomenon of stock market return volatility. Tang, Tang, and Sheng (2009) proposed a wavelet kernel function SVM, namely WSVM. The model was experimented using actual data to verify the applicability and effectiveness of the method. ANN is also widely used to solve the stock price prediction problem. ANN was used for stock price prediction (Guresen, Kayakutlu, & Daim, 2011; Montenegro & Molina, 2019; Rather, Agarwal, & Sastry, 2015). Hochreiter and Schmidhuber (1997) proposed the long short-term memory model that can accept time series continuity information and has good adaptability to time series data. At present, it has been used by many scholars to predict the volatility of stock price. Kim and Won (2018) put forward the GEW-LSTM model based on the combination of LSTM and GARCH for predicting the volatility of stock prices. Volatility can be used to further predict stock prices. All the MSE, MAE, and HMSE show that GEW-LSTM is better than the advanced forecasting methods. Ramos-Pérez, Alonso-González, and Núñez-Velázquez (2019) stacked multiple machine learning methods such as random forest, support vector machine, and artificial neural network into a combined model.

Due to the requirement of stock prediction models for interpretability. A number of scholars have conducted research on the construction of interpretable stock prediction models. Ito, Minami, Imajo, and Nakagawa (2020) proposes a novel evolutionary model. The algorithm works by aggregating several simple weak algorithms with interpretability. The method determines the importance of each weak algorithm in order to construct interpretability. Neuro-fuzzy systems are more interpretable than artificial intelligence algorithms due to their linguistic rule-based inference process. Many scholars have conducted research on constructing interpretable stock prediction models based on Neuro-fuzzy model. Cao, et al. (2020) combines Neuro-fuzzy systems and evolutionary algorithms. With the help of evolutionary algorithms to improve the prediction accuracy of Neuro-fuzzy systems. Xie et al. (2021) proposed a new 5-layer NFS model structure by combining neuro-fuzzy systems with Hammerstein–Wiener. The inference process is explained by fuzzy linguistic rules. Too many fuzzy inference rules can lead to a decrease in interpretability. Rajab and Sharma (2019) proposes to control the number of fuzzy rule bases using performance criteria to ensure the interpretability of the prediction model. Rajab and Sharma (2019) focuses on the trade-off between interpretability and accuracy.

For the interpretability of stock prediction algorithms, we want to know which features have a greater impact on the prediction results. And we hope to adjust the input features and further optimize the

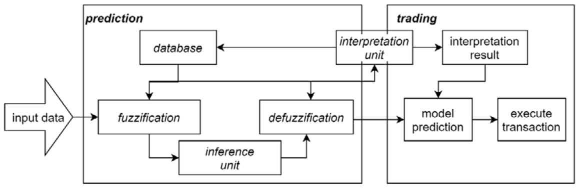

Fig. 1. Block diagram of interpretable intuitionistic fuzzy inference model.

prediction model by using the interpretability results. Common feature optimization algorithms include the Principal Component Analysis (PCA). PCA method screens for valid features byfLayer 3 — encoding lay calculating the variance of the features.

Partial dependency plots Molnar, et al. (2021) and Individual Conditional Expectation observe how the prediction results change when feature values are changed by fixing the remaining features and only observing the effect of changes in one feature on the results.

However, stock price prediction models have unique prediction accuracy requirements. For example, a prediction result of −0.1, 0.1 versus 0.5, 0.7 would both have an error of 0.2 in an ordinary accuracy. However, the error degree should not be judged to be the same for both. For stock price prediction problems, the former predicts the wrong direction and the latter predicts the right direction. The existing interpretable methods focus on the influence of features on the prediction results, and the influence of features is not exactly the same as the importance of features in the stock prediction task. The interpretive method based on intuitionistic fuzzy sets proposed in this thesis draws on the features of intuitionistic fuzzy sets and will calculate the contribution to the prediction result membership, non-membership and hesitation degrees separately, allowing for better calculation of feature importance.

3. Interpretable intuitionistic fuzzy inference model

3.1. The proposed model

The block diagram of the proposed interpretable intuitionistic fuzzy inference model is shown in Fig. 1. The proposed model can be divided into prediction part and trading part according to its functions.

The input data will go through the steps of fuzzification and defuzzification in the prediction part to form a prediction result and pass it to the trading part. In this model, the fuzzification and defuzzification methods are constructed based on data sets. Different from the traditional NFS model, which relies on knowledge to construct fuzzification methods. This will help reduce the impact of uncertainty from knowledge. The fuzzified data are presented in the form of intuitionistic fuzzy sets, which will then be serialized into input suitable for the inference unit. The rise and fall of the stock are the label to be predicted. They will be transformed into fuzzy sets by the fuzzification method.

The output of the inference unit can get the prediction result through defuzzification. According to the prediction results, stock trading decisions can be made. Based on this, a simulation experiment is conducted in this paper.

In the inference unit, we use various different types of artificial intelligence fitting algorithms and select appropriate hyperparameters to achieve more accurate fitting.

The interpretation unit obtains interpretable results by masking the input data and comparing different outputs. Through the interpretability unit, the importance of different features can be obtained, and the specific contribution of each feature in each prediction. This can be used to interpret results and optimize feature selection.

3.2. Architecture of IIFI

The architecture of the IIFI is a six-layer network, as shown in Fig. 2. The first layer consists of input nodes, which is responsible for receiving and preprocessing the input data, and then passing them to the next layer. The second layer is fuzzification layer. It receives input data from the first layer, and then transforms the crisp data into fuzzy sets. Layer 3 is in encoding layer. It builds an intuitionistic fuzzy set for each feature via fuzzy sets from layer 2. All information of each feature will be displayed with intuitionistic fuzzy sets.

Layer 4 is the inference and interpretation layer, which has two paths. The first path contains the inference module, which receives intuitionistic fuzzy sets from layer 3 and uses the inference model to calculate the fuzzy set result about the predicted label and pass it to the next layer. It can use any artificial intelligence algorithm as the fitting algorithm. The second path connects layer 3 and layer 6 and calculates the contribution degree by masking the intuitionistic fuzzy sets information from the third layer. Obtain the importance of each feature and the interpretability of the predicted results.

Layer 5 is the consequent layers. It is composed of fuzzy sets. It contains multiple comment dimensions about the labels, and each dimension has a fuzzy number representation. Layer 6 is defuzzification layer, which has two functions. It will score each stock based on the fuzzy sets information received by layer 5. At the same time, it will convert the fuzzy sets information into intuitive fuzzy sets and pass it to the interpretation block, like layer 3. The interpretation for each prediction result will also be obtained from the interpretation module in this layer.

The input data is denoted as $X = [X_1, X_2, \ldots, X_I]$, where I is the dimension of input data and also represents the number of features. And each X_i has n dimensions which means the number of stocks in the target stocks pool. The desired output and predicted output are denoted as $D = [D_1, D_2, \ldots, D_r], Y = [Y_1, Y_2, \ldots, Y_r]$, where r is the number of output dimensions. The r also means the number of fuzzy sets used to describe the value of stock. The interpretable output data used to decide how to buy or sell stock is recorded as $O = [O_1, O_2, \ldots, O_n]$, where n means the number of stocks. every stock in the target pool will calculate a crisp score denoted as $C = [C_1, C_2, \ldots, C_n]$. The score ranking determines which stocks will be bought. Each node in layer 2 and layer 5 represents a fuzzy set. E in layer 3 and O in layer 6 represent an intuitionistic fuzzy set.

In layer 2, the number of fuzzy sets corresponding to each feature is indefinite. We use $J = [J_1, J_2, \ldots, J_i]$ to describe the number of fuzzy sets to each feature. In layer 5, r is the number of fuzzy partitions to label. The detailed description of each layer is described as follows.

3.2.1. Layer 1 — input layer

In layer 1, nodes are designed to receive input data and perform data preprocess. In this layer, outliers will be corrected to maximum values. Values with too large data will be calculated logarithmically. After the preprocessing, the data will be passed to the next layer. Using f_i^1 and o_i^1 to define the input and output of layer 1 ith node I_i, which are given as:

Net-input: $f_i^1 = x_i$

Net-output: $o_i^1 = preprocess\left(f_i^1\right)$

3.2.2. Layer 2 — fuzzification layer

Layer 2 is the fuzzification layer, which converts the crisp input data into a fuzzy set through a membership function. Each node in this layer represents a fuzzy set and has a semantic expression. Each node in this layer has an individual membership function and has a semantic expression such as very large, good, bad. The core content of the fuzzification step is to construct membership function. The common methods to determine the membership function can be divided into two categories, one is to rely on the expert knowledge to define directly, the other is to determine the membership function by mining the

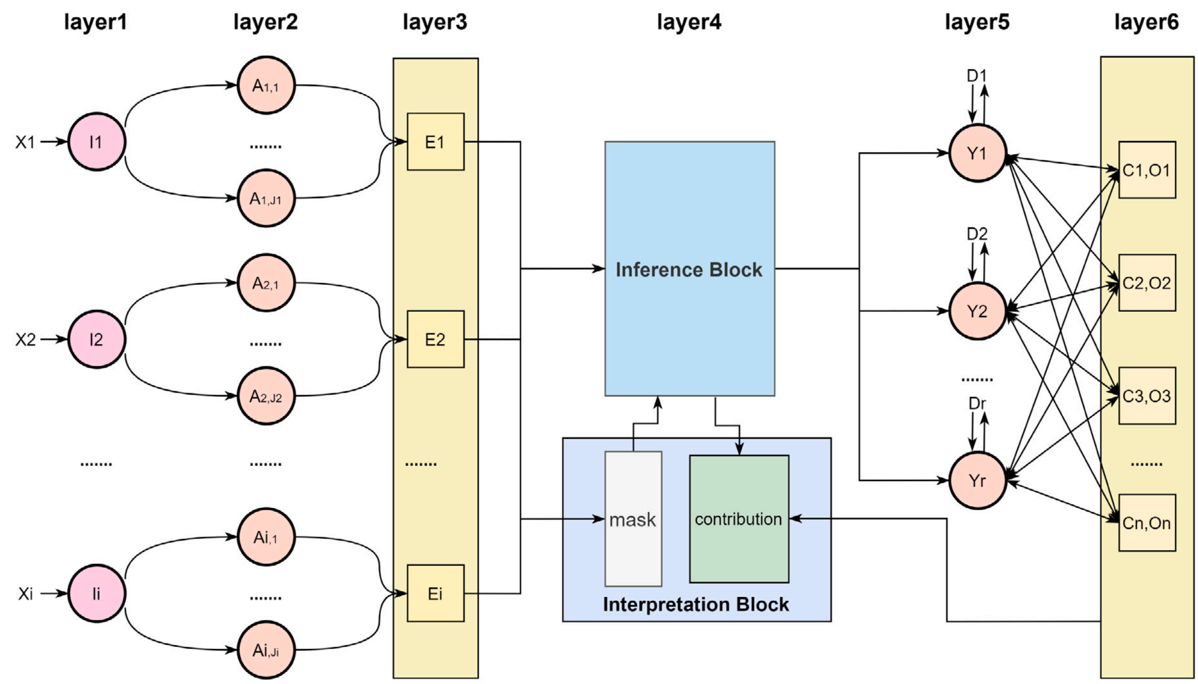

Fig. 2. Architecture of interpretable intuitionistic fuzzy inference model.

relevant information of the data. We use the second kind of method. We use fuzzy distribution method and fuzzy c-means method (FCM) to determine the membership function according to the characteristics of the features.

3.2.2.1. Fuzzy distribution method. This method is different from the traditional statistics method, which generally refers to determining the occurrence probability of random events through many experiments. The implementation process of the fuzzy distribution method is to select a fixed element U_0 in U. U is a domain of discourse. For a variable boundary set A, counting the number of times U_0 belongs to A in N times experiments. For the variable boundary set A, his boundary is not fixed in each experiment. We use A^* to represent the variable set A in each experiment. The membership frequency of U_0 to A can be calculated. The stable membership frequency is called membership degree as describe in (1):

$$A\left(u_0\right) = \lim_{n\to\infty} \frac{\text{number of } u_0 \in A^*}{n} \tag{1}$$

In this model, we divide the domain of features into multiple intervals according to the data distribution. Each interval has multiple pieces of data. Based on the different semantic sets to which the labels corresponding to each data belong, the membership frequencies of the intervals corresponding to the different semantic sets are calculated. Then the Gaussian function is used for fitting. We can uniquely determine a Gaussian function by determining the center and variance. We use c as the center of Gaussian function. $sigma$ is variance of the Gaussian function. Generally, $sigma$ is also named as the width of the Gaussian function. The membership functions are described in (2):

$$\mu(x; c, \sigma) = e^{-\frac{(x-c)^2}{\sigma^2}} \tag{2}$$

3.2.2.2. Fuzzy c-means method. Cluster analysis is a method to study the classification of things in mathematical statistics. But in fact, the classification of reality is fuzzy. The same is true of stock price prediction, so it is more reasonable to use fuzzy clustering analysis to classify problems with fuzziness. FCM (Bezdek, Ehrlich, & Full, 1984) is a fuzzy clustering method.

The data set to be clustered denoted as $X = X_1, X_2, \dots, X_n$. n is the number of data sets. The cluster center to be calculated is defined as $V = V_1, V_2, \dots, V_c$. c is the number of specified cluster centers.

The target function is described in (3):

$$J_m(U, V) = \sum_{i=1}^{n} \sum_{j=1}^{c} u_{ij}^m d_{ij}^2 \tag{3}$$

The membership value matrix is denote as (4):

$$U = \begin{pmatrix} u_{11} & \cdots & u_{1c} \\ \vdots & \ddots & \vdots \\ u_{n1} & \cdots & u_{nc} \end{pmatrix}_{nxc} \tag{4}$$

where

u_{ij} is the membership value of X_i to V_j.

d_{ij} is the distance of X_i and V_j.

m represents the parameter of the degree of fuzzification. m is generally set to 2.

The final classification result can be calculated by minimizing $J_m(U, V)$. After determining the membership value matrix U, we will use the Gaussian function for fitting. The membership value matrix U will be transformed into c membership functions.

By calculating the ratio of the sum of inter-class distance to the sum of intra-class distance L(c), the optimal number of clusters can be determined. L(c) is described in (5):

$$L(c) = \frac{\sum_{j=1}^{c} \sum_{i=1}^{n} u_{ij}^m \left\| v_j - \bar{x} \right\|^2 / (c-1)}{\sum_{j=1}^{c} \sum_{i=1}^{n} u_{ij}^m \left\| x_i - v_j \right\|^2 / (n-c)} \tag{5}$$

$\bar{x}$ represents the sample center in (6):

$$\bar{x} = \frac{\sum_{j=1}^{c} \sum_{i=1}^{n} u_{ij}^m x_i}{n} \tag{6}$$

In layer 2 Each feature corresponds to multiple fuzzy sets. Each fuzzy set corresponds to an evaluation dimension, and the membership function is used to calculate the membership value. Input data will be converted into membership value, which indicates the degree to which the input data belongs to the corresponding fuzzy set. Node A_{ij} denote the jth fuzzy set of ith dimensions of input data. Each dimension may have a different number of fuzzy sets, which is determined according to the meaning of the data and the data distribution. We unify all the membership functions in the form of Gaussian functions. We refer to the center and width of the Gaussian function as the center and width of the membership function. The layer 2 input and output are given as:

Net-input: $f_{ij}^2 = o_i^1$

Net-output: $o_{ij}^2 = \mu(f_{ij}^2, c_{ij}, \sigma_{ij})$ where

μ is the membership function.

c_{ij} is the center of membership function in μ.

σ_{ij} is the width of membership function in μ.

3.2.3. Layer 3 — encoding layer

Layer 3 is in charge of encoding. How to use fuzzy sets information effectively has always been the focus of the neuro-fuzzy system. Atanassov (2016) proposed Intuitionistic fuzzy set (IFS). IFS is one of the most influential expansion and development of Zadeh fuzzy set. FS theory can express fuzzy concepts with unclear evaluation boundaries. However, to express richer info, IFS introduced the concept of non-membership value. Further, it can express the state of not correspond to two fuzzy sets, which is the concept and degree of neutrality. IFS can describe the vague nature of the objective world in more detail and has also received a lot of attention. In layer 3, we design a conversion operator to convert multiple fuzzy sets (FS) of each feature into a single IFS.

An IFS can express more information than a FS. Therefore, we cannot directly convert FS to IFS. In layer 3, for each feature, multiple membership degree of FSs are received from the layer 2, so we propose a method to transform from FS to IFS. We use the membership function $\mu_A(x)$ to calculate the membership value of x to set A to describe a FS in layer 2. A couple of FSs can form a comment set, and they can describe features from multiple semantic concepts. We can use$[\mu_1, \mu_2, \dots, \mu_{j_i}]$ to represent the comment set of ith feature like node $A_{i,1}, \dots, A_{i,j_i}$.

IFS consists of membership function, non-membership function and hesitation function, and always denote as $[\mu(x), \nu(x), \pi(x)]$. Each node E_i in layer 3 represents an IFS. They will merge FSs info from the layer 2. Each Fuzzy set will have their membership function which has a cluster center and cluster range. In layer 3, for each feature we will choose a main positive fuzzy set and a main negative fuzzy set. If the fuzzification method is FCM, we usually use the numerical value as the criterion of the main FS. Because the features associated with stock prices generally have numerical significance. The features of stock prices include the amount of capital inflows, the range of ups and downs, etc. The magnitude of the numerical value has some connection to the result, so we use the numerical value as a criterion to determine the main FS. The maximum and minimum values will be selected as the main positive fuzzy set and the main negative fuzzy set. If the fuzzification method is Fuzzy distribution, we use the best and worst semantic set corresponding to the label as the main positive fuzzy set and the main negative fuzzy set. Because for any feature i, $\sum_{\mathrm{j}=1}^{J_i} \mu_{ij} \le 1$ and $c_1 < c_j < c_{J_i} (1 < j < J_i)$ are satisfied. So our calculation results are satisfied with the rules of IFS.

The layer 3 input and output are given as:

Net input: $f_i^3 = [\mu_1^2, \mu_2^2, \dots, \mu_{J_i}^2]$

Net output: $o_i^3 = [\mu_i^3, v_i^3, \pi_i^3]$

where $\mu_i^3(x) = \sum_{\mathrm{j}=1}^{J_i} \left(\frac{c_j - \mathrm{c}_1}{c_{J_i} - \mathrm{c}_1}\right)^2 \mu_{ij}^2$ $\upsilon_i^3(x) = \sum_{j=1}^{J_i} \left(\frac{\mathrm{c}_j - \mathrm{c}_{J_i}}{\mathrm{c}_1 - \mathrm{c}_{J_i}}\right)^2 \mu_{ij}^2$ $\pi_i^3(x) = 1 - \mu_i^3(x) - \upsilon_i^3(x)$

μ_i^3 is the membership value on the intuitionistic fuzzy set of ith dimensions of input data.

v_i^3 is the non-membership value on the intuitionistic fuzzy set of ith dimensions of input data.

π_i^3 is the hesitancy value on the intuitionistic fuzzy set of ith dimensions of input data.

c_1 is the center of the main negative fuzzy set.

c_{J_i} is the center of the main positive fuzzy set.

c_j is the center of other fuzzy sets.

3.2.4. Layer 4 — inference and interpretation layer

The fourth layer contains inference and interpretation modules. In the inference module the intuitionistic fuzzy set information transmitted by the upper layer is received and serialized as input. The output of this module is the membership value of each stock in multiple label's fuzzy sets. For example, the output is the membership values of the fuzzy set of up and down semantics for each stock. We evaluate a stock from multiple dimensions through fuzzy sets with semantic meanings such as rising and falling. In the process of training, the module will learn and fit according to the data and information of the training set, and the real value used for fitting comes from the membership value of each fuzzy set calculated by the real stock data. In this part, we need a fitting algorithm to fit the input and output data. We use a variety of different types of artificial intelligence algorithms as the fitting model of the reasoning module. These include Deep Neural Networks, LightGBM, XGBoost and Stacking (which is an ensemble learning method). They are all excellent regression algorithms.

The inference module input and output are given as:

Net input: $f^4 = [o_1^3, o_2^3, \dots, o_i^3]$

Net output: $o^4 = [Y_1, Y_1, \dots, Y_r]$ where

Y_i is the ith fuzzy set of stock fluctuations.

The interpretability module relies on the meaning of intuitionistic fuzzy sets to construct interpretability. Different from the traditional rule-based if-then interpretation. Because the input and output in the form of intuitionistic fuzzy sets in layer 3 and layer 6 concretize meaning. We adjust the hesitation of the input intuitionistic fuzzy set to 1 feature by feature. The importance of features can be obtained through the changes in the membership value, non-membership value and hesitation of the outputs. And for each individual prediction result, the contribution of each feature can be obtained. When the number of features increases, the amount of calculation of this method increases linearly. In the traditional interpretation based on inference rules, the amount of calculation increases exponentially.

3.2.5. Layer 5 — consequent layer

Layer 5 is consequent layer. Each node in the consequent layer represents a fuzzy set. This layer will use fuzzy sets to describe the stock fluctuations. Their corresponding semantics are very high, high, normal, low, very low. In the training process, the stock fluctuation data in the training set data will be converted into membership values and passed to the layer 4 as fitting data denoted as $D = [D_1, D_2, \dots, D_r]$. r represents the number of fuzzy sets corresponding to the stock fluctuations. In the prediction process, this layer will get the membership value of each stock on multiple fuzzy sets. These membership values will constitute the comment set denoted as $Y = [Y_1, Y_2, \dots, Y_r]$ and be passed to the next layer.

The layer 5 input and output are given as:

Net input: $f^5 = Y = o^4$

Net output: $o^5 = o^4 + [\mu_1^y + \mu_2^y + \dots + \mu_\mathrm{r}^y] = [\mu_1^4, \mu_2^4, \dots, \mu_\mathrm{r}^4] + [c_1^y, \sigma_1^y] + [c_2^y, \sigma_2^y] + \dots + [c_r^y, \sigma_\mathrm{r}^y]$ where

μ_i^4 is the prediction membership value of ith fuzzy set of stock fluctuations.

μ_i^y is the membership function of ith fuzzy set of stock fluctuations building by training set.

c_i^y is the center of membership function in μ_i^y.

σ_i^y is the width of membership function in μ_i^y.

3.2.6. Layer 6 — defuzzification layer

Layer 6 is defuzzification layer. The fuzzy set information received by the fifth layer needs to be defuzzified before it can be used for transaction decision-making. It will calculate the crisp value of the stock based on the predicted membership value of each fuzzy set obtained from the fifth layer. We used the center of area defuzzification method (Patel & Mohan, 2002) to calculate the crisp value as the score

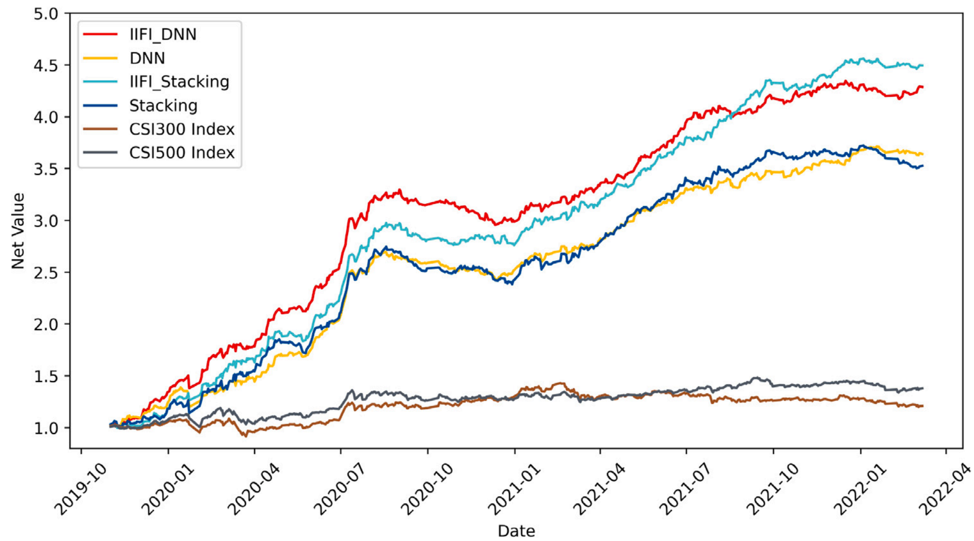

Fig. 3. Comparison of algorithms and algorithms embedded in IIFI.

of the stock. The score will be used as the basis for judging whether to purchase stocks. The scores are denoted as $O = [O_1, O_2 \ldots, O_n]$ in Fig. 2.

This layer also converts the fuzzy sets data obtained from the upper layer into intuitionistic fuzzy set data, as the layer 3 does. In Fig. 2, these intuitionistic fuzzy sets are denoted as $C = [C_1, C_2 \ldots, C_n]$. The intuitionistic fuzzy set data will be passed to the layer 4 for interpretability construction.

The layer 6 input and output are given as:

Net input: $f_j^6 = [\mu_i^4, c_i, \sigma_i]$

Net output:

$$o_j^6 = C_j^6 + O_j{}^6$$

$$C_j^6 = \frac{\sum_{i=1}^r \mu^4{}_i * c_i * \sigma_i}{\sum_{i=1}^r c_i * \sigma_i} \quad j \text{ represents the } j\text{th stock}$$

$$O_j{}^6 = \left[\mu_j^6, \upsilon_j^6, \pi_j^6\right]$$

μ_i^4 is the prediction membership value of ith fuzzy set of stock fluctuations.

c_i is the center of membership function in μ_i^y.

σ_i is the width of membership function in μ_i^y.

C_j^6 is the score of jth stock used for trading decision.

$[\mu_j^6, v_j^6, \pi_j^6]$ is the membership value, non-membership value and hesitation of jth stock about the semantics of rising.

4. Experimental results

4.1. Dataset

In this section, the performance of IIFI is evaluated on China A-Stock Market. We select the constituent stocks of CSI 300 and CSI 500 as the stock pool. CSI 300 consists of 300 of the largest and most representative securities in China A-Stock Market. CSI 500 consists of the top 500 stocks in China A-Stock Market excluding the CSI 300 constituents. They reflect large-cap stocks and mid- and small-cap stocks, respectively. These are two important indexes in Chinese A-stock market, which can express the overall trend of Chinese A-stock market. Their constituent stocks are a group of valuable and liquid stocks in Chinese A-stock market, which is of representative significance.

The data we use include Level-2 market data, financial data, etc. Level-1 and level-2 refer to different fine-grained stock market data. Level-1 market data includes real-time stock prices and trading volume. The level-2 market data contains per transaction data, per order data, 10-position order data, order volume, buy and sell queues. The level-2 market data is more informative.

We get these data from data providers, and the time range of the data is from 2017 to 2020. Data from 2017–2019 as the training set and validation set, and data from 2020 as the test set. Based on the experience of financial practitioners, we have selected 21 features, all of which are calculated from basic data processing. Appendix shows the details of the calculation of these 21 features.

4.2. Baseline methods and the application of IIFI on stock prediction

IIFI structure is a six-layer network structure, in which layer 4 can accommodate different artificial intelligence algorithms as fitting algorithms to build inference modules. To verify the effectiveness and robustness of IIFI in predicting stock profits. We embed the existing artificial intelligence algorithms into the IIFI and apply them to the stock prediction task to detect the improvement brought by IIFI. We use Dynamic Evolving Neural-fuzzy Inference System (DENFIS) (Kasabov & Song, 2002) and Neural Fuzzy Hammerstein–Wiener Model (NFHW) (Xie et al., 2021) as baseline model. Both are Neuro fuzzy inference model and suitable for stock prediction scenarios. We select existing artificial intelligence algorithms, including LightGBM (Sun et al., 2020), XGBoost (Basak, et al., 2019), Deep Neural Network (DNN) (Montenegro & Molina, 2019) and Stacking (Ramos-Pérez et al., 2019) as the fitting algorithm to embed IIFI. Lightgbm and XGBoost are decision tree-based methods. Stacking method is an ensemble learning method. We represent the combination of the fitting method and IIFI in the form of IIFI-method. They will accept the same input and predict the fluctuation of the stock. Layer 2 of IIFI relies on the data of pre 100 trading days to construct the intuitionistic fuzzy set calculation function related to the real-time market. We wanted to verify the validity of the stock prediction model on medium frequency trading. Combining the trading rules of the Chinese A-stock market with the advice of financial practitioners, we have determined the stock selection strategy. Each stock prediction strategy will select 5 stocks with the highest expected gains at 9:40 on each trading day and sell them at the opening of the next trading day. We have back-tested based on the trading rules of the Chinese A-share market.

4.3. Experimental result

As shown in Fig. 3, we list the equity curve of some algorithms under simulated trading. Detailed data is in Table 1. Compared with the original method, the IIFI-methods has a significant improvement. As shown in Table 1, compared with the original method, the algorithm

Table 1
Experimental results of algorithms and algorithms embedded in IIFI.

Method	Baseline annualized rate of return	IIFI annualized rate of return	Profitability improvement	Baseline drawdown	IIFI drawdown	Drawdown decline
DNN (Montenegro & Molina, 2019)	75.62%	88.87%	**17.52%**	13.74%	10.46%	23.88%
XGBoost (Basak, et al., 2019)	57.92%	79.23%	**36.78%**	16.15%	7.96%	**50.74%**
Stacking (Ramos-Pérez et al., 2019)	72.93%	**92.81%**	27.26%	13.43%	**7.31%**	45.57%
LightGBM (Sun et al., 2020)	**78.82%**	87.10%	10.50%	**10.22%**	11.27%	−10.28%

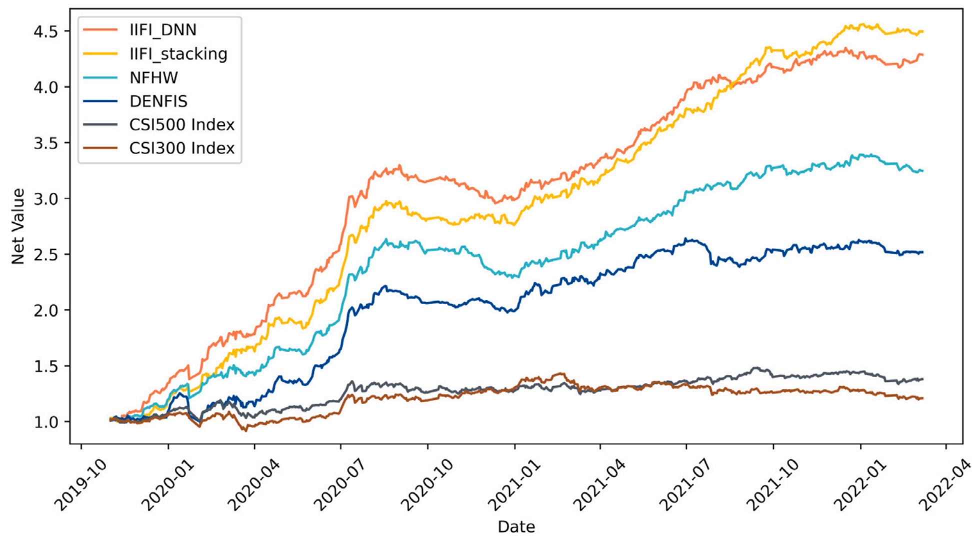

Fig. 4. Comparison of IIFI-method and baseline model.

Table 2
Experimental results of algorithms and algorithms embedded in IIFI.

Method	Net value	Annualized rate of return	Maximum drawdown	Sharpe
NFHW (Xie et al., 2021)	3.184977	66.36%	13.35%	0.637764
DENFIS (Kasabov & Song, 2002)	2.458179	48.46%	21.28%	0.501360
IIFI_DNN	4.251559	88.87%	10.46%	0.734203
IIFI_Stacking	**4.45625**	**92.81%**	**7.31%**	**0.814552**

embedded in IIFI improves the profit by more than 20% and reduces the maximum drawdown.

As shown in Fig. 4, we list the net value curve of IIFI-method and baseline model under simulated trading. Detailed data is in Table 2. IIFI-method has the best performance in terms of Profitability, maximum drawdown, and Sharpe ratio. Applicability and effectiveness are verified.

4.4. Interpretation of prediction result

IIFI can adapt to different fitting algorithms and give an interpretable expression of feature importance. We choose IIFI-Stacking as a representative. The prediction result of each stock obtained in layer 6 contains 3 elements, membership value, non-membership value, and hesitation. The membership values can be interpreted as the degree of support for the rise of stocks, and the non-affiliation values can be understood as the degree of rejection of the rise of stocks. The hesitation represents the fuzzy part of the prediction. The smaller the value, the more accurate the prediction result.

Fig. 5(a) shows the top 15 features of the change in the average membership value of rising stocks after each feature is masked. If the feature is masked, the average membership value for rising stocks rises. This indicates that the probability of actual rising stocks being predicted to rise is elevated. This shows that the masked feature is bad. Removing this feature can increase the probability that rising stocks are predicted to rise. It should be noted that if the member value of a real rising stock has fallen does not indicate that this is a good feature. The decline in membership value may be due to the increase in hesitation.

Fig. 5(b) shows the top 15 features of the hesitation changes of all stocks after the features are masked. When features were masked, the increase in hesitation indicated that the masked features carried more effective information. After the features were masked, the information content decreased, which led to an increase in hesitation. So we can assume that if the average hesitation of a feature rises after it is masked, it means that he is a good feature.

According to Fig. 5, better features and worse features can be obtained.

We use IIFI-stacking as the model method to delete two poor features and two better features respectively. Re-train the model, predict and simulate trading. The equity curve is shown in Fig. 6. According to Table 3, the model with poor features removed is better than the original model, and the model with better features removed from the maximum drawdown, profitability, and Sharpe ratio. This verifies the result of feature importance interpretation.

Fig. 7 shows the contribution of each feature in the prediction of individual stocks. Fig. 7(a)(b)(c) are three rising stocks and their prediction results are rising. Fig. 7(d)(e)(f) are three falling stocks and their prediction results are falling. The right side of 7 shows the relationship between the horizontal axis number and the corresponding feature, which is also all the features used in this article. We calculate the membership values non-membership values and hesitation values about the stock by masking each feature separately and calculate their differences from the original values. Subsequently, the ratio of the difference value of each feature to the sum of the difference values of all features is calculated. The degree of contribution of the features to the

Table 3
Comparison of experimental results.

Method	Net value	Annualized rate of return	Maximum drawdown	Sharpe
IIFI_better	**4.57146**	**94.99%**	**5.52%**	**0.921605**
IIFI	4.45625	92.81%	7.31%	0.814552
IIFI_worse	4.04835	84.85%	7.41%	0.742699

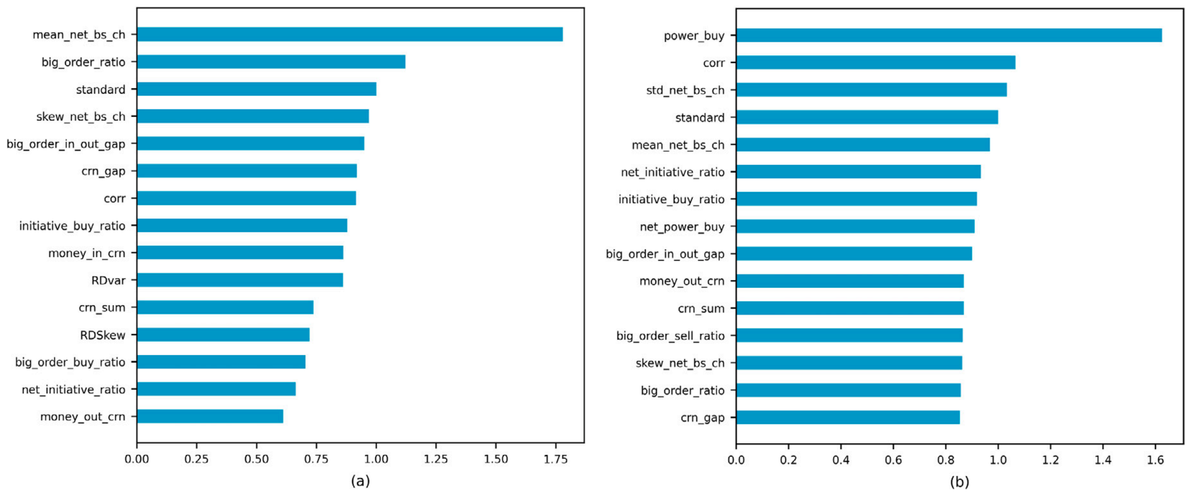

Fig. 5. Features importance (a) the top 15 features of the change in the average membership value of rising stocks (b) the top 15 features of the hesitation changes of all stocks.

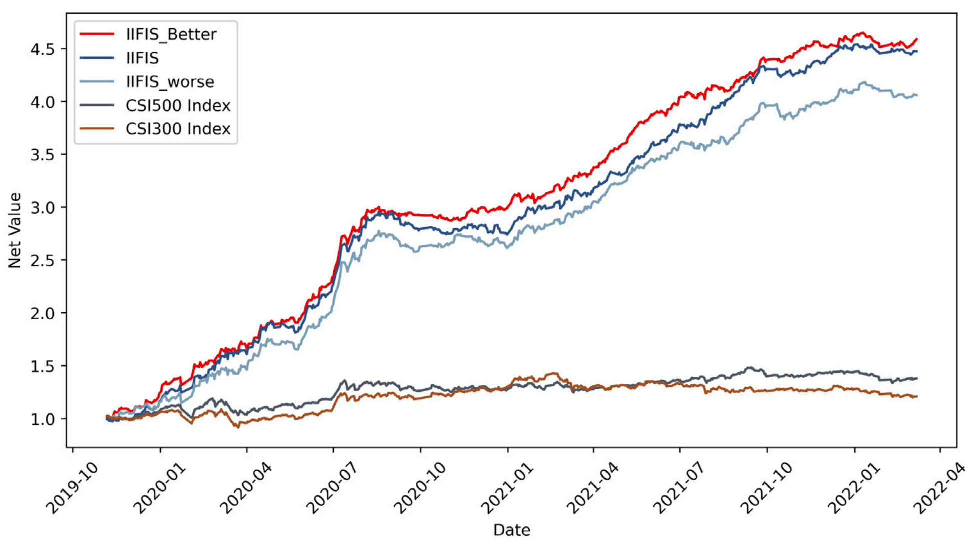

Fig. 6. Comparison between the model after feature selection and the original model.

prediction results is obtained. When predicting the rising result and the falling result, the contribution of each feature is significantly different. However, when the predicted results are all rising, the contributions of various features of different stocks are similar. This is a reasonable phenomenon.

The interpretability of the features will help financial industry personnel to conduct further analysis and indicate the contribution of each feature in each prediction.

5. Conclusion

In this paper, we propose IIFI, an interpretable intuitionistic fuzzy inference model for stock prediction, which is a six layers network structure model. On the one hand, the model solves the dependence of the traditional fuzzy model on expert information and constructs the membership function in a data-driven way to solve the problems of fuzzification and defuzzification. At the same time, the intermediate node based on intuitionistic fuzzy set is used to solve the problem that the intermediate node of neuro-fuzzy system increases exponentially with the increase of the number of features. On the other hand, it solves the problem of the lack of interpretability of black box models such as artificial intelligence algorithms. The artificial intelligence algorithms are integrated into the IIFI model, and the interpretable information is output based on intuitionistic fuzzy set with the help of interpretable module. Different from the "if-then" rule of fuzzy systems, the contribution degree is used to express the importance of features, which gives clear results when the number of features is large. Therefore, the proposed IIFI model not only retains the prediction accuracy of artificial intelligence algorithm but also increases the interpretable expression. The experimental results show that IIFI is superior to artificial intelligence algorithms and neuro fuzzy algorithms in many indexes such as income and drawdown. And the feature importance

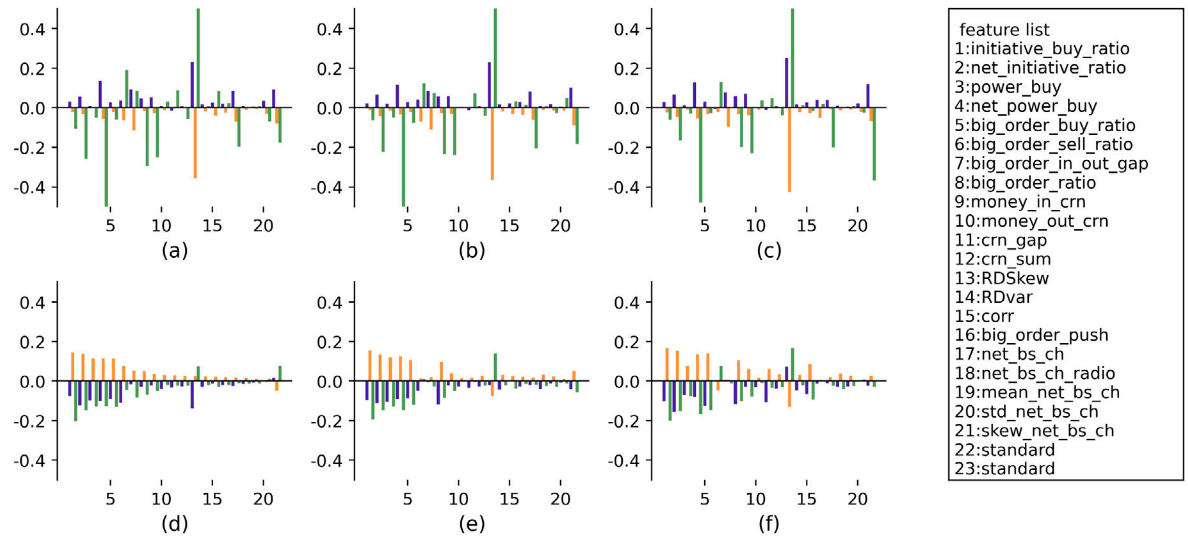

Fig. 7. The contribution of each feature of 6 individual stocks. Dark blue represents membership value, orange represents non-membership value, and green represents hesitation.

Table 4
The feature used in the experiment is listed below. These 21 features are selected from the research report of Haitong Securities and calculated from the basic data.

	Feature name	Description
1	initiative_buy_ratio	$\frac{1}{T}\sum_{n=t}^{n=t-T+1}\frac{\sum_{j\in 9:30-9:40}\text{active buy turnover}_{i,j,n}}{\sum_{j\in 9:30-9:40}\text{turnover}_{i,j,n}}$ i, j, and n respectively represent the ith stock, the jth tick, and the nth trading day. T is 5.Same below.
2	net_initiative_ratio	$\frac{1}{T}\sum_{n=t}^{n=t-T+1}\frac{\sum_{j\in 9:30-9:40}\textit{net active buy turnover}_{i,j,n}}{\sum_{j\in 9:30-9:40}\textit{turnover}_{i,j,n}}$ net active buy turnover i,j,n = active buy turnover $_{i,j,n}$ - active sell turnover $_{i,j,n}$
3	power_buy	$\frac{1}{T}\sum_{n=t}^{n=t-T+1}\frac{\text{mean}_{j\in 9:30-9:40}(\text{active buy turnover}_{i,j,n})}{\text{std}_{j\in 9:30-9:40}(\text{active buy turnover}_{i,j,n})}$
4	net_power_buy	$\frac{1}{T}\sum_{n=t}^{n=t-T+1}\frac{\text{mean}_{j\in 9:30-9:40}(\text{net active buy turnover}_{i,j,n})}{\text{std}_{j\in 9:30-9:40}(\text{net active buy turnover}_{i,j,n})}$
5	big_order_buy_ratio	$\frac{1}{T}\sum_{n=t}^{n=t-T+1}\frac{\text{big active buy order turnover}_{i,n}}{\text{turnover}_{i,n}}$
6	big_order_sell_ratio	$\frac{1}{T}\sum_{n=t}^{n=t-T+1}\frac{\text{big active sell order turnover}_{i,n}}{\text{turnover}_{i,n}}$
7	big_order_in_out_gap	$\frac{1}{T}\sum_{n=t}^{n=t-T+1}\frac{\text{big active buy order turnover}_{i,n}-\text{big active sell order turnover}_{i,n}}{\text{turnover}_{i,n}}$
8	big_order_ratio	$\frac{1}{T}\sum_{n=t}^{n=t-T+1}\frac{\text{big order turnover}_{i,n}}{\text{turnover}_{i,n}}$
9	money_in_crn	$\frac{\sum_{k=1}^{N_{i,t}}\text{active buy turnover}_{i,t,k}^{2}}{\text{turnover}_{i,t}^{2}}$
10	money_out_crn	$\frac{\sum_{k=1}^{N_{i,t}}\text{active sell turnover}_{i,t,k}^{2}}{\text{turnover}_{i,t}^{2}}$
11	crn_gap	$\frac{\sum_{k=1}^{N_{i,t}}\text{active buy turnover}_{i,t,k}^{2}}{\text{turnover}_{i,t}^{2}}-\frac{\sum_{k=1}^{N_{i,t}}\text{active sell turnover}_{i,t,k}^{2}}{\text{turnover}_{i,t}^{2}}$
12	crn_sum	$\frac{\sum_{k=1}^{N_{i,t}}\text{active buy turnover}_{i,t,k}^{2}}{\text{turnover}_{i,t}^{2}}+\frac{\sum_{k=1}^{N_{i,t}}\text{active sell turnover}_{i,t,k}^{2}}{\text{turnover}_{i,t}^{2}}$
13	RDSkew(high frequency skewness)	$\frac{1}{T}\sum_{n=t}^{n=t-T+1}\frac{\sqrt{N}\sum_{j=1}^{N}r_{i,j,n}^{3}}{\left(\sum_{j=1}^{N}r_{i,j,n}^{2}\right)^{1.5}}$
14	RDvar(percentage of downside volatility)	$\frac{1}{T}\sum_{n=t}^{n=t-T+1}\frac{\sqrt{N}\sum_{j=1}^{N}r_{i,j,n}^{2}\cdot I_{r_{i,j}<0}}{\sum_{j=1}^{N}r_{i,j,n}^{2}}$
15	corr(correlation)	$\frac{1}{T}\sum_{n=t}^{n=t-T+1}\text{corr}\left(\text{Close}_{i,j,n},\frac{\text{Vol}_{i,j,n}}{\sum_{j}\text{Vol}_{i,j,n}}\right)$
16	big_order_push	$\prod_{n=t}^{n=t-T+1}\left(\text{prod}\left(1+r_{i,j,n}\cdot I_{\{j\in IdxSet\}}\right)\right)-1$ represents the serial number of the K-line with the largest 30% of the average single transaction amount on day j
17	net_bs_ch (Net buy order's change)	$\sum_{j=1}^{k}\text{buy order's change}_{j,t}^{T}-\sum_{j=1}^{k}\text{sell order's change}_{j,t}^{T}$ i,k,T represent stock i will use the top-k orders pending order data on T day
18	net_bs_ch_radio(Net buy order's change ratio)	$\frac{\text{net buy order's change}_{k,t}^{T}}{\text{Circulating Shares}_{T}}$
19	mean_net_bs_ch(mean buy order's change ratio)	$\text{mean}\left(\text{net buy order's change ratio}_{k,t}^{T}\right)$
20	std_net_bs_ch(fluctuation ratio of buy order's change ratio)	$\text{std}\left(\text{net buy order's change ratio}_{k,t}^{T}\right)$
21	skew_net_bs_ch(skewness of buy order's change ratio)	$\text{skewness}\left(\text{net buy order's change ratio}_{k,t}^{T}\right)$

can be calculated for feature selection. For each prediction result, the contribution degree of the feature can also be obtained. This will contribute to further model optimization. In the future, we will explore the following directions.

1. Interpretable information can further optimize our model. We will explore how to use interpretability information more effectively to improve the performance of the model.
2. In addition to time series trading information, text data such as news and audio are also data sources often used in stock forecasting in recent years. IIFI, as an extension of fuzzy method, is a potential multimodal method. Incorporating more external information into the model will help to improve the effectiveness of the model.

CRediT authorship contribution statement

Weiming Wang: Data curation, Investigation, Writing – original draft. **Weiwei Lin:** Conceptualization, Methodology, Formal analysis. **Yiming Wen:** Data curation, Software. **Xiaozheng Lai:** Writing – review & editing, Investigation. **Peng Peng:** Software, Writing – review & editing, Investigation. **Yi Zhang:** Software. **Keqin Li:** Writing – review & editing, Investigation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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Appendix

See Table 4.

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