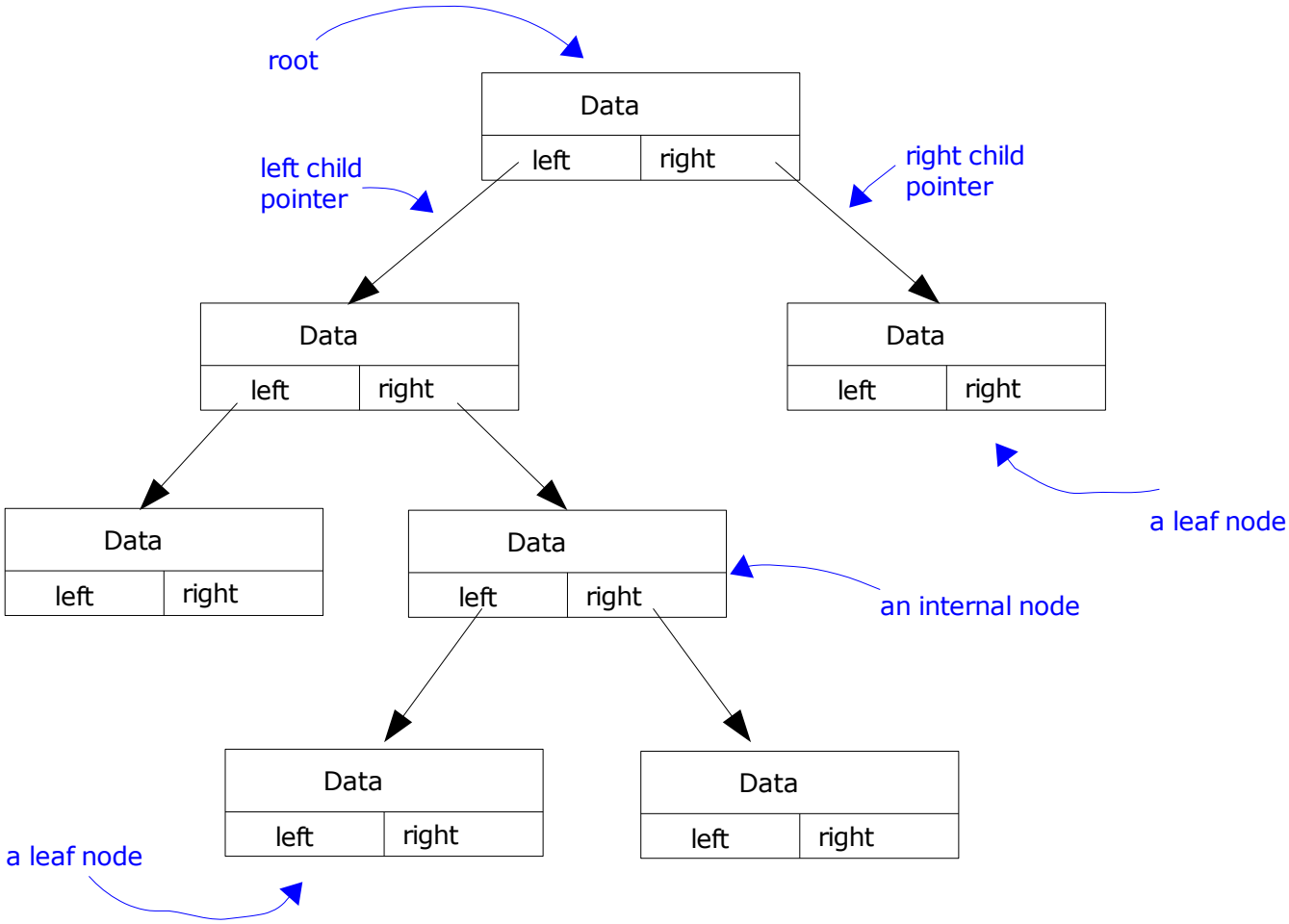
Binary Trees

A binary tree is made of nodes, where each node contains a "left" pointer, a "right" pointer, and a data element. The "root" pointer points to the top-most node in the tree. The left and right pointers recursively point to smaller "subtrees" on either side. A *null root pointer* represents a binary tree with no elements -the empty tree.

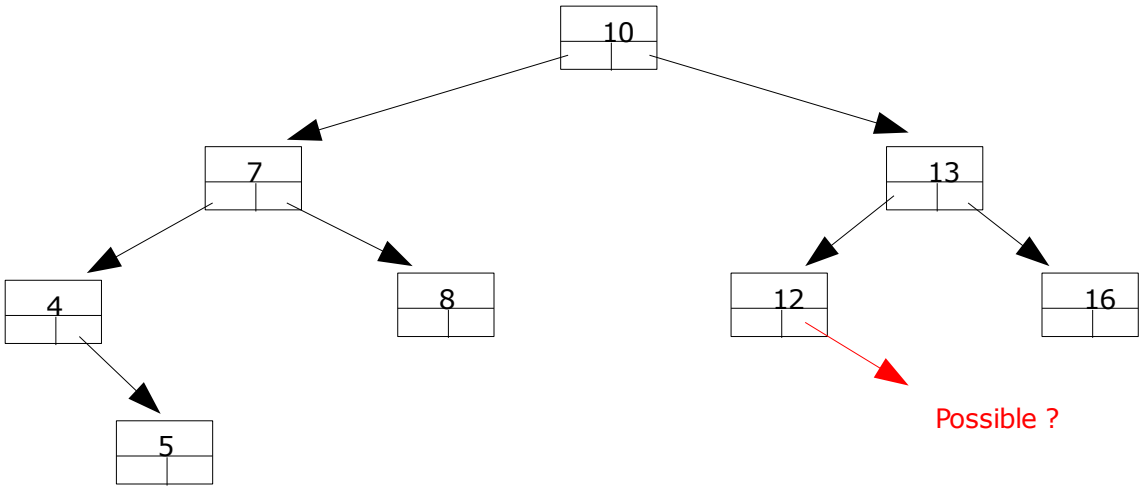


In C++, you a binary tree is built with nodes of the type:

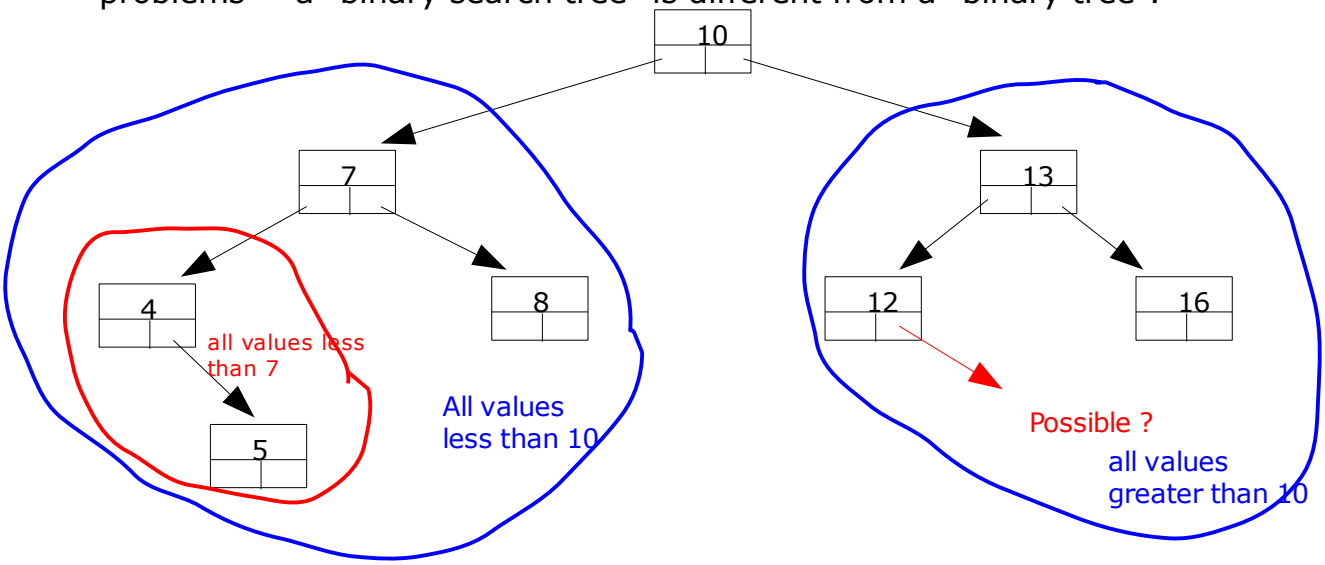
```
struct node {
    int data;
    struct node* left;
    struct node* right;
};
```

A "**binary search tree**" (BST) or "ordered binary tree" is a type of binary tree

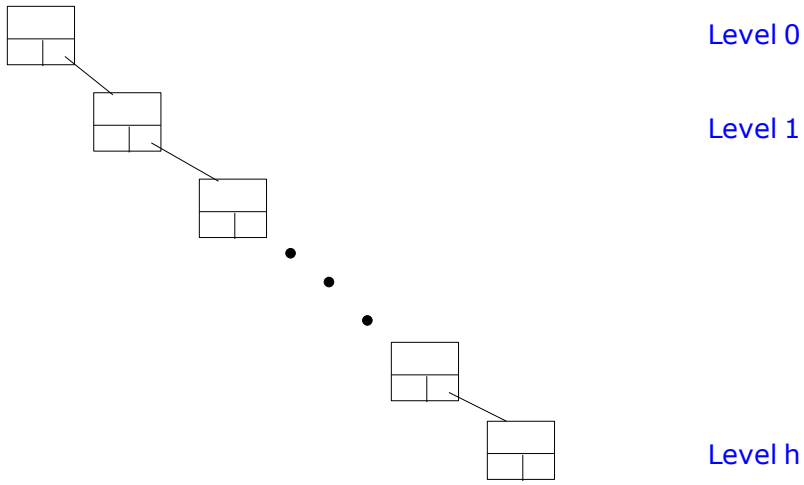
where the nodes are arranged in order: for each node, all data elements in its left subtree are less than or equal to the node (<=), and all the elements in its right subtree are greater than the node (>). Note that comparison is made between the data elements in the node.



The tree shown above is a **binary search tree** -- the "root" node is a 10, and its left subtree nodes (5, 4, 7, 8) are <= 10, and its right subtree nodes (12, 13, 16) are > 10. Recursively, each of the subtrees must also obey the binary search tree constraint: in the (5, 4, 7, 8) subtree, the 7 is the root, and its left subtree has values (4, 5) which are less than 7. Watch out for the exact wording in the problems -- a "binary search tree" is different from a "binary tree".

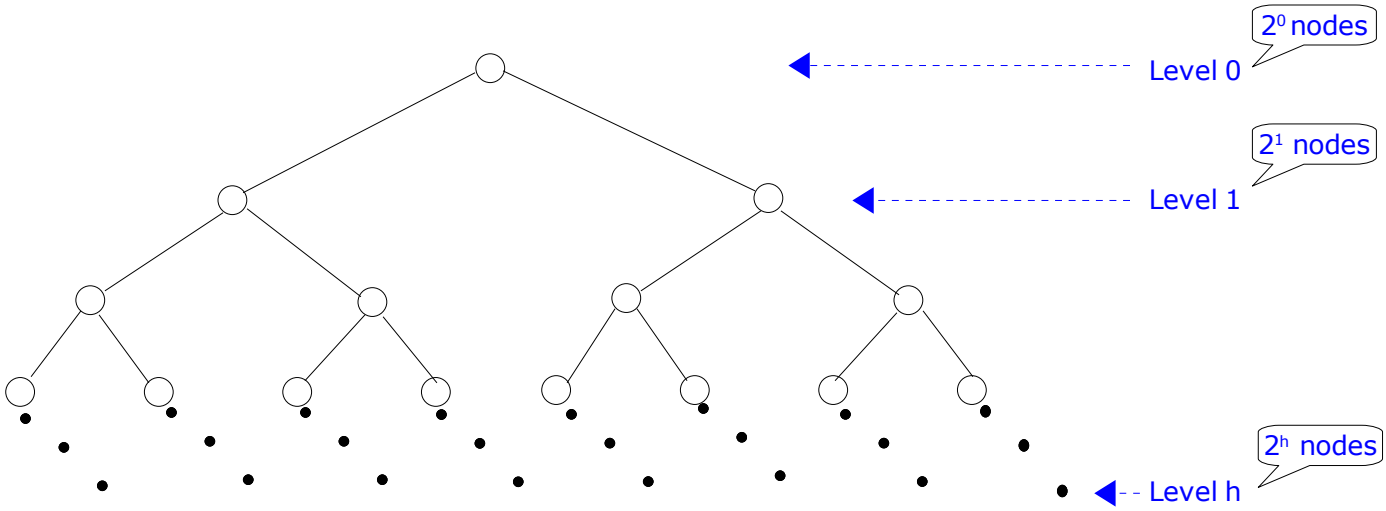


Consider a Binary Search Tree that has *n* nodes. In one extreme case, the tree could look like



This "degenerate" tree has *n-1* levels (counting root at level 0).

On the other hand consider a Binary tree that is "tightly packed", with every possible node filled in:



In this case Level 0 has 2^0 nodes, Level 1 has 2^1 nodes, ... , Level h has 2^h nodes, so that

$2^0 + 2^1 + \ldots + 2^h = n \Rightarrow 2^{h+1} - 1 = n \Rightarrow h = \log_2(n+1) - 1$

For n=1000, we see that in the "tightly packed" case, the number of levels, h is: $h=\log_2(1001)-1=8.9672$. So it is sufficient to have 9 levels to pack 1000 nodes if we use "tight packing".